WATER QUALITY MONITORING OF THE LITANI RIVER

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ABSTRACT

Water quality of the Litani river has been investigated for chemical contamination. In particular, nitrate, acidity, and salinity, have been studied through time and space. A State-Space model has been fitted to provide an on-line prediction.

Keywords: the Litani water shed, water quality, monitoring, Kalman filter

INTRODUCTION

In water quality monitoring of a watershed, measurements are often taken at regular intervals of time (hourly, daily, weekly, *etc.*) for each of the sampling sites. Various factors enter into play while choosing sampling sites: accessibility, water utilization, vicinity to urban or industrial centers, financial considerations, *etc*. The common objective in water quality monitoring is to follow the evolution of certain crucial component of the watershed and have it modeled for future prediction and control (Eigbe *et al.*, 1998). Another issue of interest is the optimality of the monitoring network and calibration of the monitoring stations that are to be installed along the river. This manuscript reports on the status of the current water quality of the Litani river in its initial stretch.

THE LITANI WATERSHED

The region along the upper course of the river was once a system of interconnected wetlands, but which is now almost dry except for a stream in the Baalbek district. The Litani river starts to form in the Shamistar Housh El-Rifka area, and is then fed by major springs further downstream in the Zahle district. It flows south in the Bekka valley until it reaches the Karaoun Lake . Some sections of the river bed are practically dry during most of the dry season (June-September) because of excessive water pumping for agriculture. The Litani also received a considerable amount of agricultural and domestic discharges including discharges from sewers. There are a number of industrial plants on the river banks that discharge their effluent directly into the river . This study focuses on this latter aspect in particular chemical contamination of the river.

STATISTICAL ANALYSIS

The interdependence of sites necessitates the use of models that can describe the components and give online estimation of such components. The situation here fits the settings of state space representation, which is widely used in many engineering applications such as tracking objects in space. One viable model to describe the contaminant level is the following:

$$Y_{tjk} = \mu_{tj} + \varepsilon_{tjk} \tag{1}$$

Where the time index t ranges from 1 to n, the site index j ranges from 1 to s, k represents replicate measurements taken on each site and ranges between 1 and m, μ_{tj} denotes the unknown level at time t and site j, Y_{tjk} is what can be observed, and errors ε_{tjk} are assumed to be a random sequence of independent and identically distributed with mean zero and variance σ_ε^2. The interdependence of sites can be described by one step Markovian process which may be represented as follows:

$$\mu_{tj} - \mu_{.j} = \theta(\mu_{t,j-1} - \mu_{.,j-1}) + v_{tj} \tag{2}$$

Where $\mu_{.j}$ is an unknown overall average property of site j, θ is an unknown 'trend' parameter, and v_{tj} is a sequence of independent and identically distributed random variables with mean 0 and unknown variance σ_v^2. Also, it is sensible to assume that errors ε_{tjk} and v_{tj} are independent for all t, j, and k.

If errors ε_{tjk} and v_{tj} are further assumed to follow the normal distribution the

$\hat{\mu}_{tj}$, the best mean square linear predictor, of μ_{tj} based on observations $Y_{1j1}, \ldots, Y_{njk}$, and $\tilde{\mu}_{tj}$, the best mean square linear estimator of μ_{tj} based on observations $Y_{1j1}, \ldots, Y_{njk}$ can be determined recursively using the Kalman filtering algorithm (Brockwell and Davis, 1990; Harvey, 1991).

The Kalman filtering algorithm requires the parameters in equations (1) and (2) to be known. If they were not known, then a non-linear optimization algorithm would be applied to estimate the parameters. Here we propose a new methodology based on the maximum likelihood estimator of the parameters before the recursive steps of the Kalman filtering algorithm are applied. To obtain the likelihood function, $L(Y;\Theta)$ where $\Theta = (\theta,\ \sigma_v^2,\ \sigma_\varepsilon^2)'$, define vectors

$\tilde{Y}_{t1k} = \left(Y_{t1k} - \mu_{.1}, \ldots, Y_{tsk} - \mu_{.s}\right)'$

and

$\tilde{Y}_{t0k} = \left(Y_{t0k} - \mu_{.0}, \ldots, Y_{t,s-1,k} - \mu_{.s-1}\right)'$

Where $Y_{t0k} = 0, \mu_{.0} = 0$, and V' denotes the transpose of vector V. Now, the vector quantity $\tilde{Y}_{t1k} - \theta\tilde{Y}_{t0k}$ follows a multivariate normal distribution with mean vector 0 = $(0, \ldots, 0)'_{s\times 1}$ and covariance matrix $\Sigma_{s\times s}$ which is given by Toeplitz $\left(\left[\sigma_\upsilon^2 + \sigma_\varepsilon^2\left(1+\theta^2\right)\right], -\theta\sigma_\varepsilon^2, 0, \ldots, 0\right)$. Thus the likelihood function, L_{tk} (Y; Θ), for a fixed time t and replicate k will be given by

$$L_{tk}\ (Y;\ \Theta) = (2\pi)^{-s/2}|\Sigma|^{-1/2}\exp - \frac{1}{2}\left(\tilde{Y}_{t1k} - \theta\tilde{Y}_{t0k}\right)'\ \Sigma^{-1}\ \left(\tilde{Y}_{t1k} - \theta\tilde{Y}_{t0k}\right), \qquad (3)$$

And hence the likelihood equation for $t = 1, \ldots, n$ and $k = 1, \ldots, m$ becomes

$$L(Y;\Theta) = \coprod_{t=1}^{n} \prod_{k=1}^{m} L_{tk}\ L(Y;\Theta). \qquad (4)$$

It is worth noting here that Σ can be written as $M\Lambda M'$ where

$M = (\sin(\pi ij/(s+1)) / ((s+1)/2)^{1/2})^{s}_{i,j=1}$, $\Lambda = \mathrm{diag}\left(\lambda_j\right)_{j=1}^{s}$

And

$\lambda_j = \sigma_\upsilon^2 + \left(1+\theta^2\right)\sigma_\varepsilon^2 - 2\sigma_\varepsilon^2\theta\cos\left(\pi j/(s+1)\right)$.

Hence an explicit expression of the likelihood becomes available and can be maximized by numerical methods such as the Newton-Raphson algorithm (Kennedy and Gentle, 1982).

MODEL FITTING

When maximizing the likelihood function in (4), it is easy to see that

$$\hat{\mu}_{.j} = \frac{\sum_{t=1}^{n} \sum_{k=1}^{m} Y_{tjk}}{2n} \text{ here } j = 1, \ldots, s. \tag{5}$$

Parameters θ, σ_{υ}^2, and σ_{ε}^2 are yet to be estimated and may be obtained by the Newton-Raphson algorithm. The following gives the maximum likelihood estimates of the parameters for the collected data:

1. Figure 1 gives the nitrate levels across all sites. The standard error for each $\hat{\mu}_{.j}$ is given in parenthesis.

TABLE 1

Average Nitrate Concentration Across Sites

Site	1	2	3	4	5
$\hat{\mu}_{.j}$	4.95(1.80)	4.27(1.43)	3.24(1.73)	3.15(1.27)	3.15(1.56)

And estimates of system parameters are: $\hat{\theta} = .81$, $\sigma_{\upsilon}^2 = .26$, and $\sigma_{\varepsilon}^2 = .25$

Figure 1. Nitrate concentration data for the five sites.

2. Figure 2 gives the pH values of the monitoring sites.

TABLE 2

pH Averages Across Sites

Site	1	2	3	4	5
$\hat{\mu}_{.j}$	7.61(.20)	7.51(.26)	7.26(.29)	7.33(.34)	7.35(.42)

and system parameters are: $\hat{\theta} = 0.325$, $\sigma_{\upsilon}^{2} = 0.000$, and $\sigma_{\varepsilon}^{2} = 0.05$

Figure 2. pH level data for the five sites.

3. For the EC the following results were obtained (Figure 3).

TABLE 3

Average Electric Conductivity Across Sites

Site	1	2	3	4	5
$\hat{\mu}_{.j}$	0.79(.23)	1.07(.36)	0.84(.36)	0.87(.30)	0.85(.28)

And system parameters are: $\hat{\theta} = 0.10$, $\sigma_{\upsilon}^{2} = 0.00$, and $\sigma_{\varepsilon}^{2} = 0.015$.

Figure 3. EC data for the five sites.

It is evident from the above results that nitrate level is showing a slow downward trend through sites. On the other hand, neither pH nor EC levels seem to remain random throughout all the sites.

As a final remark, it is quite complicated nowadays to come up with continuous reliable data.. However, steps may be taken to regulate over pumping, and dumping of domestic effluent and industrial waste water directly in the river or its tributaries. Moreover, it is recommended to test the water for contaminants in order to avoid any public health problems that might arise from such contamination since the waters of the Litani are used by many sectors.

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REFERENCES

Brockwell, P, and Davis, R., 1990. *Time series: theory and methods*. Springer-Verlag, New York, inc.

Eigbe, U., Beck, M.B., Wheater, H.S. and Hirano, F. 1998. Kalman filtering in groundwater flow modelling: problems and prospects. *Stochastic Hydrology and Hydraulics*, 12: 15-32.

Harvey, A. 1991. *Forecasting, structural time series models and the Kalman filter*. Cambridge University Press, Cambridge, UK.

Kennedy, W. and Gentle, J. 1982. *Statistical computing*. Dekker, New York.