ON INTERSECTION GRAPH OF INTUITIONISTIC FUZZY SUBMODULES OF A MODULE

B. Talaee[1], G.Nasiri[2]

[1]Department of Mathematics, Faculty of Basic Sciences,
Babol Noshirvani University of Technology, Shariati Ave., Babol, Iran.
[2]Department of Mathematics, Faculty of Basic Sciences,
Babol Noshirvani University of Technology, Shariati Ave., Babol, Iran.
behnamtalaee@nit.ac.ir

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ABSTRACT

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There are some interesting relations between submodules of a module and its intuitionistic fuzzy (IF) submodules. In this paper we investigate some relationships between submodules of a module and its IF submodules. Then we introduce a graph structure on IF submodules of a module and obtain some properties of it, that is the main goal of this paper. We define the intersection graph of submodules of a module M (G) and we show that a submodule N of M is a center in $G_M - \theta$ if and only if χ_N^{IF} is a center in $G - \chi_\theta^{IF}$. We get some relationships between IF submodules of a module and their supports, as vertices of IF graph and crisp graph of a module M, respectively. We show that an IF submodule A of M is center in IF graph of M if and only if A° is a center in crisp graph of M.

In prime ring R, we show that every vertex of intersection graph of IF ideals of R is center. In general the nature of intersection graph of IF submodules of a module under intersection, homomorphic images, finite sum and other algebraic operations of its vertices, are investigated.

Keywords: Fuzzy submodule, Intuitionistic fuzzy submodule, Intersection graph of IF submodules, Center of IF graph.

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INTRODUCTION

After the introduction of fuzzy sets by L. A. Zadeh [18], a number of applications of this fundamental concept have come up.

A. Rosenfeld [16] was the first one to define the concept of fuzzy subgroups of group. C. V. Negoita and D. A. Ralescu [14] applied this concept to modules and defined fuzzy submodules of a module. A. Rosenfeld [16] interpreted the concept of fuzzy group which has been influencing the researchers gradually. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by K. T. Atanassov in [3]. Using this idea, B. Davvas [8] established the intuitionistic fuzzification of the concept of submodules of a module. The intersection graph of algebraic structures has been studied by several authors. J. Bosak [4] in 1964 defined the graph of semigroups. Inspired by his work, B. Csakany and G. Pollak [7] in 1969, studied the graph of subgroups of a finite group. Recently, in 2009, the intersection graph of ideals of a ring, was considered by Chakrabarty, Ghosh, Mukherjee and sen [5]. Rajkhowa, K. K. and Saikia, H. K in [15] study on center of intersection graph of submodules of a module. Here we define the intersection graph of intuitionistic fuzzy submodules of a module. Our main goal is to study the connection between the algebraic properties of a module and the graph theoretic properties of the graph associated to it. In this paper after some essential preliminaries of fuzzy sets and intuitionistic fuzzy sets and submodules, we study the center of intersection graph of intuitionistic fuzzy submodules of a module and establish some results relating with corresponding crisp concepts. This intersection graph of intuitionistic fuzzy submodules is an infinite graph. The importance of intuitionistic fuzzy theory is that it improve fuzzy theory such that the non-membership of each member is a value between 0 and 1 – its membership value. Intuitionistic fuzzy set is very profitable model to elaborate uncertainty and vagueness involved in decision making. Intuitionistic fuzzy set has many applications in sciences and industry such as medical diagnosis, medicine, decision making problems.

For more information about intuitionistic sets and systems, readers are urged to refer to the following literature [6, 9, 11, 13].

MATERIAL AND METHODS

A background of intuitionistic fuzzy submodules

Throughout this paper R will denote a ring with identity and all modules are unitary left R-modules. Since then we use briefly "IF" for "intuitionistic fuzzy". In this section first we give some basic definitions of F and IF sets. We refer the reader to [3, 8, 18] for these definitions.

Note that we use notations $\vee$ and $\wedge$ for maximum and minimum, repectively.

Definition 1: Let X be a set. A map $\mu : X \to [0,1]$ is called a $fuzzy\, subset$ of X. The collection of all fuzzy subsets of X is denoted by $[0,1]^X$.
Let $\alpha \in [0,1]^X$, then
1. $\alpha \subseteq \beta$ if and only if $\alpha(x) \leq \beta(x)$ for every $x \in [0,1]$;
2. $(\alpha \cup \beta)(x) = \alpha(x) \vee \beta(x)$ for every $x \in [0,1]$;
3. $(\alpha \cap \beta)(x) = \alpha(x) \wedge \beta(x)$ for every $x \in [0,1]$.

Definition 2: A fuzzy set μ of a ring R is called a $fuzzy\, ideal$, if it satisfies the following properties:
1. $\mu(x-y) \geq \mu(x) \wedge \mu(y)$;
2. $\mu(xy) \geq \mu(x) \vee \mu(y)$,for all $x, y \in X$.

Definition 3: A fuzzy subset μ of a module M is called a $fuzzy\, submodule$ of M if for every $x, y \in M$ and $r \in R$, the following conditions are satisfied
1. $\mu(0) = 1$;
2. $\mu(x+y) \geq \mu(x) \wedge \mu(y)$;
3. $\mu(rx) \geq \mu(x)$.

We use the notation $F(M)$ for the set of all fuzzy submodules of the module M .
Let $\alpha, \beta \in F(M)$. Then the sum of α and β is defied by
$$(\alpha+\beta)(x) = \vee\{\alpha(y) \wedge \beta(y) \mid y+z = x; y, z \in M\}$$
for every $x \in M$.

Definition 4: An $intuitionistic\ fuzzy\, set$ (in short IFS) A of a non-void set X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote respectively the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

Definition 5: Let X be a non-void set and $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B)$ be $IFS's$ of X. Then
1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
2. $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$;
3. $A^c = (\nu_A, \mu_A)$ is called the IFS complement of A;
4. $A \cap B = \{(x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)); x \in X\}$;

5. $A \cup B = \{(x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)); x \in X\}$;

Definition 6: A IF set A=(μ_A, ν_A) of R is called an *intuitionistic fuzzy ideal* (*IF* ideal), if it satisfies the following properties:
1. $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$;
2. $\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$;
3. $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$;
4. $\nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y)$.

For two IF ideals A and B of a ring R define
$AB = (\mu_{AB}, \nu_{AB})$, such that
$\mu_{AB}(x) = \vee\{\mu_A(y) \wedge \mu_B(z) \mid yz = x\}$ and $\nu_{AB}(x) = \wedge\{\nu_A(y) \vee \nu_B(z) \mid yz = x\}$, for every $x \in R$. It is easy to see that $AB \subseteq A$, for every two IF ideals A and B of R.

Definition 7: Let M be an R-module and A=(μ_A, ν_A) an IFS of M. Then A is called an *intuitionistic fuzzy submodule* (IFM) of M, (denoted by $A \leq_{IF} M$), if A satisfies the following
1. $\mu_A(0) = 1, \nu_A(0) = 0$
2. $\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y)$, for all $x, y \in M$;
   $\nu_A(x+y) \leq \nu_A(x) \vee \nu_A(y)$, for all $x, y \in M$;
3. $\mu_A(rx) \geq \mu_A(x)$, for all $x \in M$ and $r \in R$;
   $\nu_A(rx) \leq \nu_A(x)$, for all $x \in M$ and $r \in R$.

We denote the set of all IFM of M by $IF(M)$. If A, B are two $IFM's$ of M such that $A \subseteq B$, then we say A is an IF submodule of B and denote by $A \leq_{IF} B$.

Definition 8: Let M be an R-module, $N \subseteq M$ and $\alpha \in [0,1]$. Define the IFS $\alpha_N = (\mu_{\alpha_N}, \nu_{\alpha_N})$ of M as follows

$$\mu_{\alpha_N}(x) = \begin{cases} \alpha & x \in N \\ 0 & otherwise \end{cases} \quad and \quad \nu_{\alpha_N}(x) = \begin{cases} 1-\alpha & x \in N \\ 1 & otherwise \end{cases}$$

for all $x \in M$.

If $\alpha = 1$, then $\mu_{\alpha_N} = \chi_N$ and $\nu_{\alpha_N} = \chi^c_N$, where χ_N denotes the characteristic function of N. In this case we write $\alpha_N = \chi^{IF}_N = (\chi_N, \chi^c_N)$. We denote χ^{IF}_N by 1^{IF}_N and χ^{IF}_θ by θ also.

Let $A = (\mu_A, \nu_A)$ be an IFS of M. Define
$\mu^*_A = \{x \in M \mid \mu_A(x) > 0\}$ and $\nu^*_A = \{x \in M \mid \nu_A(x) < 1\}$.
Also $\mu_{*A} = \{x \in M \mid \mu_A(x) = \mu_A(0)\}$ and $\nu_{*A} = \{x \in M \mid \nu_A(x) = \nu_A(1)\}$.
In general for every $t \in M$ define level subsets
$(\mu_A)_t = \{x \in M \mid \mu_A(x) \geq t\}$ and $(\nu_A)_t = \{x \in M \mid \nu_A(x) \leq 1 - t\}$.
If $A = (\mu_A, \nu_A) \subseteq B = (\mu_B, \nu_B)$ are two IFS' s of an R-module M, then obviously $\mu^*_A \subseteq \mu^*_B$ and $\nu^*_A \subseteq \nu^*_B$.

Let $A_* = \mu_{*A} \cap \nu_{*A}$, $A^* = \mu^*_A \cap \nu^*_A$ and so $A^\circ = \mu^*_A \cup \nu^*_A$ and $A^t = (\mu_A)_t \cap (\nu_A)_t$.
We have the following proposition.

Proposition 9:
1. If A is an IFM of M then $\mu^*_A = A^* \subseteq \nu^*_A$, $\mu_{*A} = A_*$ an $A^\circ = \nu^*_A$.
2. If A is an IFM of M then $\mu_{*A} = A_* \subseteq \nu_{*A}$, and also $A_* = M$ if and only if $A = \chi^{IF}_M$.
3. $A_* = 0$ if and only if $A = \chi^{IF}_\theta$.
4. If $A \leq_{IF} M$, then $\chi^{IF}_{A*} \subseteq A$ and $A_* \subseteq A^* \subseteq A^\circ$.
5. If $\chi^{IF}_M = A$, then $M = A^*$.
6. $\chi^{IF}_M = A$ if and only if $M = A_*$.
7. If $A \subseteq B$ are two IFS' s of M, then $A^* \subseteq B^*$, $A_* \subseteq B_*$ and $A^\circ \subseteq B^\circ$.
8. If $A = \chi^{IF}_\theta$, then $A^* = \theta$.
9. $A = \chi^{IF}_\theta \Leftrightarrow A^\circ = \theta$.

- Proof. All are easy and follow from definitions.

Definition 10: Let M, N be two R-modules and $f: M \to N$ an R-homomorphism. Let $A = (\mu_A, \nu_A) \leq_{IF} M$ and $B = (\mu_B, \nu_B) \leq_{IF} N$. Then $f(A) = (\mu_{f(A)}, \nu_{f(A)})$ and

$f^{-1}(B)=(\mu_{f^{-1}(B)},\nu_{f^{-1}(B)})$ are IFM's of N and M respectively, such that for all $y\in N$

$$(\mu_{f(A)})(y)=\begin{cases}\bigvee\{\mu_A(x)\mid y=f(x)\} & y\in Im(f)\\ 0 & y\notin Im(f)\end{cases}$$

and

$$(\nu_{f(A)})(y)=\begin{cases}\bigwedge\{\nu_A(x)\mid y=f(x)\} & y\in Im(f)\\ 1 & y\notin Im(f)\end{cases}$$

and for every $x\in M$

$$(\mu_{f^{-1}(B)})(x)=\mu_B(f(x)) \quad and \quad (\nu_{f^{-1}(B)})(x)=\nu_B(f(x)).$$

Proposition 11: *Let* M *be an* R*-module and* $N\subseteq M$ *. Then* $N\le M$ *if and only if* $\chi_N^{IF}\le_{IF} M$.

- Proof. Suppose that N is a submodule of M . Then $\theta\in N$ and hence $\chi_N(\theta)=1$ and $\chi_N^c(\theta)=0$.

Now let $x,y\in M$. If $x,y\in N$, then $x+y\in N$, so $1=\chi_N(x+y)\ge\chi_N(x)\wedge\chi_N(y)$ and $0=\chi_N^c(x+y)\le\chi_N^c(x)\vee\chi_N^\nu(y)$.

If $x\notin N$, then $\chi_N(x+y)\ge\chi_N(x)\wedge\chi_N(y)=0$ and $\chi_N^c(x+y)\le\chi_N(x)\vee\chi_N(y)=1$.

Similar to this case we get if $y\notin N$.

Now let $x\in M$ and $r\in R$. If $x\in N$, then $rx\in N$ and so we have $1=\chi_N(rx)\ge\chi_N(x)$ and $0=\chi_N^c(x)\le\chi_N^c(rx)$.

If $x\notin N$, then $0=\chi_N(x)\le\chi_N(rx)$ and also $1=\chi_N^c(x)\ge\chi_N^c(rx)$.

Therefore χ_N^{IF} is an IFM of M .

For converse suppose that χ_N^{IF} is an IFM of M . So $\chi_N(\theta)=1$ and hence $\theta\in N$. Now let $x,y\in N$ and $r\in R$, then $\chi_N(rx+y)\ge\chi_N(rx)\wedge\chi_N(y)\ge\chi_N(x)\wedge\chi_N(y)=1$. So $rx+y\in N$. That is N is a submodule of M .

Example 12: Let $M=\mathrm{Z}_{12}$ over Z and $N=2\mathrm{Z}_{12}$, $K=3\mathrm{Z}_{12}$. Then χ_N^{IF} and χ_K^{IF}

are IFM ' s of M .

Example 13: Let $M =_{\mathbb{Q}} \mathbb{R}$. Then $\mathbb{Q} \leq M$ but $\mathbb{Z}$ M. So $\chi_{\mathbb{Q}}^{IF} \leq_{IF} M$ and $\chi_{\mathbb{Z}}^{IF}$ M, by proposition 2.14.

DISCUSSION AND RESULTS

Center of Intersection graph of IF submodules of a module

In this section we use notations GR, VX and EG for graph, vertex and edge respectively.

A GR G consists of a set $V(G)$ of vertices or points and a collection $E(G)$ of pairs of vertices called edges. If a and b are two vertices of a GR and if pair (a, b) is an EG denoted by e, we say that e is an EG between a and b or a and b are near. In our discussion, all GRs are simple. The GR $H = (W, F)$ is a subGR of the GR $G = (V, E)$ if W is a subset of V and F is a subset of E. If $H = (W, F)$ is a subGR of the GR $G = (V, E)$ such that an EG exists in F between two vertices in W if and only if an EG exists in E between those two vertices, the subGR H is said to be caused by the set W, which is maximal subGR of G with respect to the set. A lane in a GR is an alternating sequence of vertices and EG $a_0 x_1 a_1 ... x_n a_n$ in which each EG x_i is $a_{i-1} a_i$. The distance of a lane is n, the number of occurrences of EG in it. A route is a lane in which all vertices are different. For vertices x and y of G, we define $d(x, y)$ to be the distance of any shortest route from x to y. G is said to be connected, if there exists a route between every pair of vertices of it, otherwise it is said to be disconnected.

Now we are going to remember the definition of intersection GR on algebraic structures. The intersection GR of ideals of a ring is a GR with VX set as the collection of nontrivial ideals of the rings such that any two vertices are near if their intersection is not zero. The intersection GR G_M of submodules of M is a GR with VX set $V(G_M)$ is the collection of all submodules of M and any two different $A, B \in V(G_M)$ are near if and only if $A \cap B \neq 0$. The notation $G_M - 0$ stands for the caused subGR of G_M which does not contain the VX 0. In the same sense, the intersection GR G of $F(M)$ is a GR with $V(G) = F(M)$ and for two different F submodules $\alpha, \beta \in F(M)$, α, β are near if and only if $\alpha \cap \beta \neq \chi_\theta$ and we write α adj β. If α and β are not near, we write α nadj β.

The intersection GR of IF submodules of a module M (denoted by G or $G_{1_M^{IF}}$ or $G_{\chi_M^{IF}}$) is a GR such that its vertices set is $IF(M)$ and two vertices A and B are near if and only if $A \cap B \neq \chi_\theta^{IF}$, while $G - \chi_\theta^{IF}$ denote the subGR of G without the VX χ_θ^{IF}. We refer to [1, 2, 4, 5, 7, 10, 12, 17] for some recently researches about some algebraic GRs and F GRs.

Definition 1: A VX of a GR G is called center if it is near with every VX of G.

Theorem 2: *Let N be a module and $N \leq M$. Then N is a center in $G_M - \theta$ if and only if χ_N^{IF} is a center in $G - \chi_\theta^{IF}$.*

- Proof. Suppose that N is a center in $G_M - \theta$. If $A = (\mu_A, \nu_A) \leq IFM$ and $A \neq \chi_\theta^{IF}$, then $A^\circ \leq M$ and $A^\circ \neq \theta$. Then $N \cap A^\circ \neq \theta$. We will prove $\chi_N^{IF} \cap A \neq \chi_\theta^{IF}$. Assume $\theta \neq x \in N \cap A^\circ$. So $\chi_N(x) = 1$ and $\mu_A(x) > 0$ or $\nu_A(x) < 1$. We can conclude from recent statement that $\chi_N(x) \wedge \mu_A(x) > 0$ or $\chi_N^c(x) \vee \nu_A(x) < 1$, implies $\chi_N^{IF} \cap A \neq \chi_\theta^{IF}$. Hence χ_N^{IF} is a center in $G - \chi_\theta^{IF}$.

Conversely, let χ_N^{IF} be a center in $G_M - \chi_\theta^{IF}$. We will show that N is a center in $G_M - \theta$. For this let $\theta \neq K \leq M$. By assumption, $\chi_N^{IF} \cap \chi_K^{IF} \neq \chi_\theta^{IF}$ and hence
there exists $\theta \neq x \in M$ such that $\chi_N(x) \wedge \chi_K(x) \neq \chi_\theta(x)$,
or
there exists $\theta \neq y \in M$ such that $\chi_N^c(y) \vee \chi_K^c(y) \neq \chi_\theta^c(y)$.
Case 1: There exists $\theta \neq x \in M$ such that $\chi_N(x) \wedge \chi_K(x) \neq \chi_\theta(x)$. Then $\chi_N(x) > 0$ and hence $\chi_N(x) = 1$. Similarly $\chi_K(x) = 1$, and so $\theta \neq x \in N \cap K$; i.e., $N \cap K \neq \theta$.
case 2: There exists $\theta \neq x \in M$ such that $\chi_N^c(x) \vee \chi_K^c(x) \neq \chi_\theta^c(x) = 1$. So $\chi_N^c(x) < 1$ and hence $\chi_N^c(x) = 0$, implies $x \in N$.
Similarly $x \in K$.

Therefore $\theta \neq x \in N \cap K$. So in both cases we conclude N is a center in $G_M - \theta$.

Corollary 3: *If N and K are two submodules of M such that $N \subseteq K$, then N is a* *center in* $G_K - \theta$ *if and only if* χ_N^{IF} *is a center in* $G_{\chi_K} - \chi_\theta^{IF}$.

Example 4: $n\mathbb{Z}$ is a center in $G_{\mathbb{Z}} - \theta$, for any $n = 1,2,...$ so $\chi_{n\mathbb{Z}}^{IF}$ is center in $G - \chi_\theta^{IF}$, where G is the intersection GR of IF submodules of $\mathbb{Z}$.

Example 5: It is clear that neither $3\mathbb{Z}_{12}$ nor $4\mathbb{Z}_{12}$ is center in $G_{\mathbb{Z}_{12}} - \theta$, so neither $\chi_{3\mathbb{Z}_{12}}^{IF}$ nor $\chi_{4\mathbb{Z}_{12}}^{IF}$ is center in $G - \chi_\theta^{IF}$. But $\chi_{2\mathbb{Z}_{12}}^{IF}$ is a center in $G - \chi_\theta^{IF}$.

Lemma 6: *If* A *and* B *are two IF ideal of* R, *then* $AB \subseteq A \cap B$.

- Proof. We should prove $\mu_{AB}(x) \le \mu_{A\cap B}(x)$ and $\nu_{AB}(x) \ge \nu_{A\cap B}(x)$, for all $x \in R$

If $x = yz$, then $\mu_{A\cap B}(x) = \mu_{A\cap B}(yz) = \mu_A(yz) \wedge \mu_B(yz) \ge \mu_A(y) \wedge \mu_B(z)$, so $\mu_{A\cap B}(x) \ge \vee\{\mu_A(y) \wedge \mu_B(z) \mid x = yz\} = \mu_{AB}(x)$.
Similarly $\nu_{A\cap B}(x) \le \nu_A(y) \vee \nu_B(z)$; for all $y, z \in R$ with $x = yz$. Hence $\nu_{A\cap B}(x) \le \wedge\{\nu_A(y) \vee \nu_B(z) \mid x = yz\} = \nu_{AB}(x)$. Thus $AB \le A \cap B$.
W
A ring R is called *prime* if θ is a prime ideal of R. An IF ideal A of R is called prime if whenever $IJ \subseteq A$ for some IF ideals I, J of R, then $I \subseteq A$ or $J \subseteq A$.

Lemma 7: *If* R *is a prime ring, then* χ_θ^{IF} *is an IF prime ideals of* R.

- Proof. Suppose that A, B are two IF ideal of R such that $AB \subseteq \chi_\theta^{IF}$. We must prove $A \subseteq \chi_\theta^{IF}$ or $B \subseteq \chi_\theta^{IF}$.

First we show that For this let $x = y_0 z_0 \in A^\circ B^\circ$ where $y_0 \in A^\circ$ and $z_0 \in B^\circ$. So $y_0 \in \mu_A^*$ or $y_0 \in \nu_A^*$ and $z_0 \in \mu_B^*$ or $z_0 \in \nu_B^*$. Since $\mu_A^* \subseteq \nu_A^*$ and $\mu_B^* \subseteq \nu_B^*$, we conclude that $y_0 \in \nu_A^*$ and $z_0 \in \nu_B^*$. So $1 > \nu_A(y_0) \vee \nu_B(z_0) \ge \wedge\{\nu_A(y) \vee \nu_B(z) \mid x = yz\} = \nu_{AB}(x)$. This implies $x = \theta$; as $AB = \chi_\theta^{IF}$. Now since R is prime, so $A^\circ = \theta$ or $B^\circ = \theta$. Finally by proposition 2.11 we get $A = \chi_\theta^{IF}$ or $B = \chi_\theta^{IF}$.

Theorem 8: *If* $M = R$ *is prime, then every VX of* $G - \chi_{\theta}^{IF}$ *is center.*

- Proof. If possibly suppose that A is not a center in $G - \chi_{\theta}^{IF}$. Then there exists a VX B in $G - \chi_{\theta}^{IF}$ with B nadj A. Then $AB \subseteq \chi_{\theta}^{IF}$, as $AB \subseteq A \cap B$ by Lemma 3.5. Since R is a prime ring so χ_{θ}^{IF} is a prime IF ideal of R by Lemma 3.6. So $A = \chi_{\theta}^{IF}$ or $B = \chi_{\theta}^{IF}$ a contradiction.

Example 9: It is well known that $\mathbb{Z}$ is a prime ring. So every VX in the intersection GR of nonzero IF submodules of $\mathbb{Z}$ is a center.

Lemma 10: *Let* A, B *be two* IFM *'s of the module* M. *Then*

1. $(A+B)^* = A^* + B^*$.
2. $(A+B)^\circ = A^\circ + B^\circ$.
3. $(A \cap B)^* = A^* \cap B^*$.
4. $(A \cap B)^\circ = A^\circ \cap B^\circ$.

- Proof. (1) Let $x \in (A+B)^* = \mu^*_{(A+B)}$, then

$$0 < \mu_{A+B}(x) = \bigvee \{\mu_A(y) \wedge \mu_B(z) \mid y + z = x; y, z \in M\}$$

So there exist $y_0, z_0 \in M$ such that $x = y_0 + z_0$ and $\mu_A(y_0) \wedge \mu_B(z_0) > 0$, implies $\mu_A(y_0) > 0$ and $\mu_B(z_0) > 0$ and so $y_0 \in \mu_A^* = A^*, z_0 \in \mu_B^* = B^*$. Hence $x = y_0 + z_0 \in A^* + B^*$. Therefore $(A+B)^* \subseteq A^* + B^*$.

In the other hand let $x \in A^* + B^*$. Then there exist $y_0 \in A^*$ and $z_0 \in B^*$ such that $x = y_0 + z_0$. Now

$$\mu_{A+B}(x) = \bigvee \{\mu_A(y) \wedge \mu_B(z) \mid y + z = x; y, z \in M\} \geq \mu_A(y_0) \wedge \mu_B(z_0) > 0$$

Hence $x \in \mu^*_{(A+B)} = (A+B)^*$. Therefore $A^* + B^* \subseteq (A+B)^*$.

(2) Let $x \in (A+B)^\circ = \nu^\circ_{A+B}$, then $1 > \nu_{A+B}(x) = \wedge\{\nu_A(x) \vee \nu_B(y) \mid y + z = x; y, z \in M\}$. Therefore there exist $x_0, y_0 \in M$ such that $x = y_0 + z_0$, $\nu_A(x_0) < 1$ and $\nu_B(y_0) < 1$

and so $x_0 \in A^\circ, y_0 \in B^\circ$. For converse suppose that $x = y_0 + z_0 \in A^\circ + B^\circ$, such that $y_0 \in A^\circ = \nu_A^\circ$ and $z_0 \in B^\circ = \nu_B^\circ$. Then $\nu_A(y_0) < 1$ and $\nu_B(z_0) < 1$ and so $1 > \nu_A(y_0) \vee \nu_B(z_0) > \wedge\{\nu_A(y) \vee \nu_B(z) \mid x = y + z; y, z \in M\} = \nu_{A+B}(x)$. This implies (3),(4) are clear.

Lemma 11: *Let* $A, B \leq_{IF} M$. *Then*

1. If $A \cap B = \chi_\theta^{IF}$, then $A^* \cap B^* = 0$.
2. $A \cap B = \chi_\theta^{IF}$ if and only if $A^\circ \cap B^\circ = 0$.

- Proof. Use proposition 2.11 and Lemma 3.9.

Theorem 12: *Let* A *be a nonzero* IF *submodule of the module* M. *Then* A *is a center in* $G - \chi_\theta^{IF}$ *if and only if* A° *is a center in* $G_M - \theta$.

- Proof. Suppose that A is a center in $G - \chi_\theta^{IF}$ and $\theta \neq N \leq M$. Then $\chi_N^{IF} \neq \chi_\theta^{IF}$

and so $A \cap \chi_N^{IF} \neq \chi_\theta^{IF}$. We will show that $A^\circ \cap N \neq \theta$. Consider two cases:
Case 1: There exists $\theta \neq x \in M$ such that $\mu_A(x) \wedge \chi_N(x) > 0$. Then $x \in \mu_A^* \subseteq A^\circ$ and ($\chi_N(x) = 1$) $x \in N$. So $x \in A^\circ \cap N$.
Case 2: There exists $\theta \neq x \in M$ such that $\nu_A(x) \vee \chi_N^c(x) < 1$. Then $x \in \nu_A^* \subseteq A^\circ$ and ($\chi_N^c(x) = \theta$) $x \in N$. So $x \in A^\circ \cap N$; i.e. A° is a center in $G_M - \theta$.
Conversely assume A° is a center in $G_M - \theta$ and $\chi_\theta^{IF} \neq B \leq_{IF} M$. Then $B^\circ \neq 0$ and hence $A^\circ \cap B^\circ \neq \theta$. Now by Lemma 3.10 and proposition 2.11, $A \cap B \neq \chi_\theta^{IF}$, as required.
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Theorem 13: *Let* $A \subseteq B$, *be two IF submodules of* M. A *is a center in* $G_B - \chi_\theta^{IF}$ *if and only if* A° *is a center in* $G_{B^\circ} - \theta$.

- Proof. Suppose that A is a center in $G_B - \chi_\theta^{IF}$. Then A° is a non-zero

submodule of M and also $A^\circ \subseteq B^\circ$.

Let N be a VX in $G_{B^\circ} - \theta$. Define an IF submodule $C = (\mu_C, \nu_C)$ of B by

$$\mu_C(x) = \begin{cases} \mu_B(x) & x \in N \\ 0 & x \notin N \end{cases} \quad \text{and} \quad \nu_C(x) = \begin{cases} \nu_B(x) & x \in N \\ 1 & x \notin N \end{cases}$$

then $C^\circ = N$.

Now for $x \in N$ we have, or $\nu_C(x) < 1$, by definition of C and $N \subseteq B^\circ$.

Since $N \neq \theta$, so there exists $0 \neq x \in N$ so that $\mu_C(x) > 0$ or $\nu_C(x) < 1$ and this implies that $C \neq \chi_\theta^{IF}$. Then $A \cap C \neq \chi_\theta^{IF}$ and so for some non-zero $y \in M$ we have $\mu_A(y) > 0$ or $\nu_A(y) < 1$ and also $\mu_C(y) > 0$ or $\nu_C(y) < 1$. Hence $0 \neq y \in A^\circ \cap C^\circ$ as desired.

Conversely assume that A° is a center in $G_{B^\circ} - \theta$ and $C \in G_B - \chi_\theta^{IF}$. Then $C^\circ \in G_{B^\circ} - \theta$ and hence $C^\circ \cap A^\circ \neq \theta$. Let $\theta \neq x \in C^\circ \cap A^\circ$. So $\mu_C(x) > 0$ or $\nu_C < 1$ and $\mu_A(x) > 0$ or $\nu_A(x) < 1$. It is not difficult to see that $\mu_C(x) \wedge \mu_A(x) > 0$ or $\nu_C(x) \vee \nu_A(x) < 1$, that implies $C \cap A \neq \chi_\theta^{IF}$ as required.

Lemma 14: *Let $\theta \neq K_1 \leq M_1 \leq M$ and $\theta \neq K_2 \leq M_2 \leq M$. If K_1 is a center in $G_{M_1} - \theta$ and K_2 is a center in $G_{M_2} - \theta$, then $K_1 \cap K_2$ is a center in $G_{M_1 \cap M_2} - \theta$.*

- Proof. By hypothesis, $K_1 \cap K_1' \neq \theta$ for every $\theta \neq K_1' \leq M_1$ and $K_2 \cap K_2' \neq \theta$ for every $\theta \neq K_2' \leq M_2$. Let $\theta \neq N \leq M_1 \cap M_2$, then $N \cap K_1 \neq \theta$. Since $N \cap K_1 \leq M_2$, so $(N \cap K_1) \cap K_2 \neq \theta$, as required.

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Lemma 15: *Let $f : M \to N$ be a module homomorphism. If $\theta \neq N_1 \leq N$ is a center in $G_N - \theta$ then $f^{-1}(N_1)$ is a center in $G_M - \theta$.*

- Proof.

Case 1: If $0 \neq M_1 \leq M$. If $f(M_1) = \theta$ then $M_1 \subseteq kerf \subseteq f^{-1}(N_1)$, so $M_1 \cap f^{-1}(N_1) = M_1$.

Case 2: If $f(M_1) \neq \theta$ then $N_1 \cap f(M_1) \neq \theta$. This means that there exists $\theta \neq y \in N_1 \cap f(M_1)$. Hence there exists $\theta \neq x \in M_1$ such that $\theta \neq y = f(x)$ and also $f(x) \in N_1$. Then $x \in f^{-1}(N_1) \cap M_1$, as required.

Theorem 16:

1. Let $A \leq_{IF} B \leq_{IF} M$. Then A is a center in $G - \chi_\theta^{IF}$ if and only if A and B are centers in $G_B - \chi_\theta^{IF}$ and $G - \chi_\theta^{IF}$, respectively.

2. Let $B, B' \leq_{IF} M$. If A and A' are centers in $G_B - \chi_\theta^{IF}$ and $G_{B'} - \chi_\theta^{IF}$, respectively, then $A \cap A'$ is a center in $G_{B \cap B'} - \chi_\theta^{IF}$.

3. Let $f: M \to N$ be a module homomorphism. If A is center in $G_{1_N^{IF}} - \chi_\theta^{IF}$, then $f^{-1}(A)$ is center in $G_{1_M^{IF}} - \chi_\theta^{IF}$.

- Proof. 1. First, we assume that A is a center in $G - \chi_\theta^{IF}$. Consider a VX C of $G_B - \chi_\theta^{IF}$, then C is also a VX of $G - \chi_\theta^{IF}$. By assumption $A \cap C \neq \chi_\theta^{IF}$, i.e. A is a center in $G_B - \chi_\theta^{IF}$. Again, if we consider a VX D of $G - \chi_\theta^{IF}$, then A adj D. Thus B is also a center in $G - \chi_\theta^{IF}$.

Conversely, suppose that A and B are centers in $G_B - \chi_\theta^{IF}$ and $G - \chi_\theta^{IF}$, respectively. Now, for a VX E of $G - \chi_\theta^{IF}$, we have B adj E, as B is a center in $G - \chi_\theta^{IF}$. Then, $0 \neq A \cap (B \cap E) \subseteq A \cap E$, since A is a center in $G_B - \chi_\theta^{IF}$. From this, it is observed that e(A)=1 in $G - \chi_\theta^{IF}$. Thus, A is a center in $G - \chi_\theta^{IF}$.

2. Assume that A and A' are centers in $G_B - \chi_\theta^{IF}$ and $G_{B'} - \chi_\theta^{IF}$, respectively. By Theorem 3.12, we have A° and are centers in $G_{B^\circ} - \theta$ and $G_{B'^\circ} - \theta$ respectively. Also using Lemma 3.13, $A^\circ \cap A'^\circ$ is a center in $G_{B^\circ \cap B'^\circ} - \theta$. Again by Theorem 3.12, it follows that $A \cap A'$ is a center in $G_{B \cap B'} - \chi_\theta^{IF}$, that completes the proof.

3. It can be easily verified that $f^{-1}(A^\circ) = (f^{-1}(A))^\circ$. Suppose that A is a center in

$G_{1_N^{IF}} - \chi_\theta^{IF}$. It follows from Theorem 3.11 that A° is a center in $G_N - \theta$. By Lemma 3.14, $f^{-1}(A^\circ)$ is a center in $G_M - \theta$. Thus again by Theorem 3.11 we get $f^{-1}(A)$ is a center in $G_{1_M^{IF}} - \theta$. Hence 3 hold.

Lemma 17: *Let* $M_1, M_2 \le M$ *such that* $M_1 \cap M_2 = \theta$. *If* $K_1 \le M_1$ *and* $\pi_1 : M_1 \oplus M_2 \to M_1$ *is the projection map, then* $\pi_1^{-1}(K_1) = K_1 \oplus M_2$.

- Proof. It is clear.

Theorem 18: *Let* $A_i \subseteq B_i$ *be IF submodules of a modules* M*, for* $i = 1,2,...,n$. *If* $\{A_i\}_{i=1}^n$ *is a disconnected set of vertices of* G*, and* A_i *is a center in* $G_{B_i} - \chi_\theta^{IF}$ *for each* $i \in \{1,2,...,n\}$*, then* $\{B_i\}_{i=1}^n$ *is also a disconnected set and* $A_1 + A_2 + ... + A_n$ *is a center in* $G_{B_1+B_2+...+B_n} - \chi_\theta^{IF}$.

- Proof. First we show that the theorem is true for $n = 2$. We consider a disconnected set $\{A_1, A_2\}$ of vertices of G such that A_1 and A_2 are centers in $G_{B_1} - \chi_\theta^{IF}$ and $G_{B_2} - \chi_\theta^{IF}$, respectively. By using Theorem 3.12, A_1° and A_2° are centers in $G_{B_1^\circ} - \theta$ and $G_{B_2^\circ} - \theta$, respectively. By Lemma 3.13, we have $A_1^\circ \cap A_2^\circ$ is a center in $G_{B_1^\circ \cap B_2^\circ} - \theta$. By hypothesis, $A_1 \cap A_2 = \chi_\theta^{IF}$ which concludes that $A_1^\circ \cap A_2^\circ = \theta$ by Lemma 3.10. Thus $B_1^\circ \cap B_2^\circ = \theta$ and from this we see that $\{B_1, B_2\}$ is also a disconnected set of vertices.

Now take the projection maps $\pi : B_1^\circ \oplus B_2^\circ \to B_2^\circ$ and $\eta : B_1^\circ \oplus B_2^\circ \to B_2^\circ$. By the Lemma 3.14 and Lemma 3.16, it can be seen that $\pi^{-1}(A_1^\circ) = A_1^\circ + B_2^\circ$ and $\eta^{-1}(A_2^\circ) = A_2^\circ + B_1^\circ$ are centers in $G_{B_1^\circ + B_2^\circ} - \theta$. We have A_1° nadj B_2° and A_2° nadj B_1°. It is not difficult to see that $A_1^\circ + A_2^\circ$ is a center in $G_{B_1^\circ + B_2^\circ} - \theta$. Hence by Lemma 3.9 (2), $(A_1 + A_2)^\circ$ is a center in $G_{(B_1+B_2)^\circ} - \theta$. So $A_1 + A_2$ is a center in $B_1 + B_2$, by Theorem 3.12. Next assume that the theorem is true for $n-1$, then $\{B_1, B_2,..., B_{n-1}\}$ is a disconnected set of vertices and

$A_1 + A_2 + ... + A_{n-1}$ is a center in $G_{B_1+B_2+...+B_n-1} - \chi_\theta^{IF}$. Now by the above case it is clear that $(B_1 + B_2 + ... + B_n - 1)$ nadj B_n and $(A_1 + A_2 + ... + A_n - 1) + A_n$ is a center in $G_{(B_1+B_2+...+B_n-1)+B_n} - \chi_\theta^{IF}$. Hence $\{B_i\}_{i=1}^n$ is also a disconnected set and $A_1 + A_2 + ... + A_n$ is a center in $G_{B_1+B_2+...+B_n} - \chi_\theta^{IF}$, as required.

Lemma 19: *Let M be an R-module and $A, B, C \leq_{IF} M$ such that $A \cap B = \chi_\theta^{IF}$ and $(A+B) \cap C = \chi_\theta^{IF}$, then $A \cap (B+C) = \chi_\theta^{IF}$.*

- Proof. Straightforward.

Theorem 20: *For any $A \in V(G)$, there is a non-near VX C to A such that $A+C$ is a center in $G - \chi_\theta^{IF}$.*

- Proof. We consider that A is a non-zero IF submodule of M. Let $\Omega = \{B \leq_{IF} M \mid A \cap B = \chi_\theta^{IF}\}$. Clearly, $\Omega \neq \emptyset$. By Zorn's lemma Ω has a maximal element C, with respect to $A \cap C = \chi_\theta^{IF}$. Thus we obtain a non-near VX C to A. Now, we show that $A+C$ is a center in $G - \chi_\theta^{IF}$. Suppose $A+C$ is not a center in $G - \chi_\theta^{IF}$. This means $e(A+C) > 1$. Then there is a non-zero $D \in V(G)$ such that $(A+C)$ nadj D, and this gives A nadj $(C+D)$. But, maximality of C with respect to $A \cap C = \chi_\theta^{IF}$ implies that $A+C = C$. Therefore, we get $D = D \cap (A+C) = \chi_\theta^{IF}$ by Lemma 3.18, which is absurd. Hence the theorem is obtained.

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Corollary 21: *Let $A \subseteq B$ be two IF submodules of M. Then for any $A \in V(G_B)$, there is a non-near VX C to A such that $A+C$ is a center in $G_B - \chi_\theta^{IF}$.*

Definition 22: Suppose that G is the intersection GR of IF submodules of a module M. Let $A \in V(G)$. Then a VX $B \in V(G)$ is said to be a *complement* of A if A nadj B and $A + B = 1_M^{IF}$. G is said to be a *complemented* GR if every VX of G has a complement.

Theorem 23: *If G is a complemented GR, then so is G_A for any IF submodule A of M.*

- Proof. Let $B \in V(G_A)$. Then $B' \cap A$ is a complement of B in G_A, where B' is a complement of B in G.

Definition 24: An IF submodule A of M is called *maximal* if A is a maximal element in the set of all non-constant IF submodules of M, with respect to the set inclusion.

Lemma 25: *(Modularity Low): Let M be a module and $A=(\mu_A,\nu_A)$, $B=(\mu_B,\nu_B)$ and $C=(\mu_C,\nu_C)$ be IFM 's of M. Then $A\cap(B+C)\supseteq(A\cap B)+(A\cap C)$. Moreover if $B\subseteq A$, then $A\cap(B+C)=B+(A\cap C)$.*

- Proof. The first statement is clear.

To see the second statement, suppose that $B\subseteq A$. Then for every $x\in M$ we have

$$(\mu_{B+(A\cap C)})(x)=\bigvee\{\mu_B(y)\wedge(\mu_A(z)\wedge\mu_C(z))\mid y+z=x;(x-y=z)\}$$

$\geq\bigvee\{\mu_B(y)\wedge(\mu_A(x)\wedge\mu_A(y))\wedge\mu_C(z)\mid y+z=x\}$ = (since $B\subseteq A$)

$\bigvee\{\mu_B(y)\wedge\mu_A(x)\wedge\mu_C(z)\mid y+z=x\}=\mu_A(x)\wedge(\bigvee\{\mu_B(y)\wedge\mu_C(z)\mid y+z=x\})$ =

$$(\mu_{A\cap(B+C)})(x)$$

Also

$$(\nu_{B+(A\cap C)})(x)=\bigwedge\{\nu_B(y)\vee(\nu_A(z)\vee\nu_C(z))\mid y+z=x;(x-y=z)\}$$

$\leq\bigwedge\{\nu_B(y)\vee(\nu_A(x)\vee\nu_A(y))\vee\nu_C(z)\mid y+z=x\}$ = (since $B\subseteq A$)

$\bigwedge\{\nu_B(y)\vee\nu_A(x)\vee\nu_C(z)\mid y+z=x\}=\nu_A(x)\vee(\bigwedge\{\nu_B(y)\vee\nu_C(z)\mid y+z=x\})$ =

$$(\nu_{A\cap(B+C)})(x)$$

So we conclude that $A\cap(B+C)\subseteq B+(A\cap C)$.

Finally $A\cap(B+C)=B+(A\cap C)$ in this case.

Lemma 26: *Let M be an R-module and A,B two IF submodules of M.*

1. For $\forall t\in(0,1]$ $A_t\subseteq B_t$ if and only if $A\subseteq B$.
2. For $\forall t\in(0,1]$ $A_t=B_t$ if and only if $A=B$.

- Proof. Straightforward.

Let $t \in (0,1]$ and $x \in X$, define the IF F point $x_t^{IF} = (\mu_t, \nu_t)$ such that

$$\mu_t(y) = \begin{cases} t & y = x \\ 0 & y \neq x \end{cases} \quad \text{and} \quad \nu_t(y) = \begin{cases} 1-t & y = x \\ 1 & y \neq x \end{cases}, \text{ for every } y \in X.$$

If A is an IF subset of X, then the notation $x_t^{IF} \in A$ means $x \in A_t$.

Theorem 3.27 *Let A be a VX in G. If G_A is a complemented GR, then there is a non-trivial maximal IF submodule of A which is not a center $G_A - \chi_\theta^{IF}$.*

- Proof. It is not sufficient to show that for $t \in (0,1]$ and for each $x_t^{IF} \neq \chi_\theta^{IF} \in A$, we have a maximal IF submodule C of A such that $x_t^{IF} \in C$. We consider $x_t^{IF} \in A$. Let $\Omega = \{B \mid B \in IF(A), x_t^{IF} \notin B\}$. It is clear that $\chi_\theta^{IF} \in \Omega$, hence $\Omega \neq \varnothing$. So by Zorn's lemma Ω has a maximal element say. We show that C is a maximal IF submodule of A. Let $C_t \subseteq D_t \subseteq A_t$. Since $D_t \subseteq A_t$, so $D \subseteq A$. As $G(A)$ is complemented, therefore there exists $D' \in IF(A)$ with $A = D + D'$ and D nadj D'. Now $D \cap (C + D') = C + (D \cap D') = C + \chi_\theta^{IF} = C$.

Thus $x_t^{IF} \notin C$ implies either $x_t^{IF} \notin D$ or $x_t^{IF} \notin (C + D')$. If $x_t^{IF} \notin D$, then $D = C$, as C is maximal with $x_t^{IF} \notin C$. So $D_t = C_t$. Also if $x_t^{IF} \notin (C + D')$, then $C + D' = C$. This gives $C_t + D_t' = C_t$. Therefore $A = D + D'$ gives Thus C is maximal with $x_t^{IF} \notin C$. From this, we get that there exists a maximal F-submodule C of A with $x_t^{IF} \notin C$ if $x_t^{IF} (\neq \chi_\theta^{IF}) \in A$. We observe that $\bigcap \{A \mid A$ is a maximal submodule of $A\} = \chi_\theta^{IF}$, as desired.

Definition 28: An IF-submodules $B = \chi_\theta^{IF}$ of M is said to be *simple* if $A \subseteq B$, where $A \in IF(M)$ implies either $A = \chi_\theta^{IF}$ or $A = B$.

Theorem 29: *Let M be a module. If 1_M^{IF} is the sum of simple IF submodules of M, then 1_M^{IF} is the only center in $G - \chi_\theta^{IF}$.*

• Proof. If possible, we assume that A is a center in $G-\{\chi_\theta^{IF}\}$ which is different from 1_M^{IF}. But 1_M^{IF} is the sum of simple IF submodules of 1_M^{IF}. Let $\{A_i\}_{i\in I}$ be the collection of all simple IF submodules of 1_M^{IF}. Then $1_M^{IF} = \sum A_i$.
Since A is center , therefore $A \cap A^\circ \neq 0$ for every i. As for every i, A_i is a simple IF submodules of M, thus $A \cap A_i \leq A_i$ gives $A \cap A_i = A_i$. That means A contains all simple IF -submodules of M. From this $1_M^{IF} \leq A$, implies $A = 1_M^{IF}$.
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Theorem 30: *If 1_M^{IF} is the only center in $G-\chi_\theta^{IF}$, then the intersection of maximal paper F -submodules of 1_M^{IF} is χ_θ^{IF}.*

• Proof. Let $A \leq 1_M^{IF}$. Then by Corollary 3.20 there is a non-near VX B with $A+B$ is a center in G. From the given condition $A+B = 1_M^{IF}$. This means that B is a complement of A. Thus G is a complement GR. Now, following the same way of Theorem 3.26, we get the result.

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