Arabic Grammar by Mathematical Principles

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ABSTRACT

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In mathematics, any equation is formed from specific elements; these elements are connected to each other according to a system of relationships (Borowski, 1995). This system allows us to build the general rule for the equation or the mathematical issue; similarly, the sentence is formed from elements that have grammatical properties, which can be transformed to mathematical data. Hence, the elements or the words that carry the grammatical properties of the sentence correlate to each other to form a mathematical relation; this mathematical relation forms the theory of sentence construction. Therefore, the analysis process is correct when we can collect mathematical data contained within the elements; in other words, we indicate the mathematical data contained in the words located in a specific sentence. These mathematical data will form the elements of the mathematical theory of the Arabic sentence.

Keywords: grammar-math, math language, word derivative.

INTRODUCTION

The elements in mathematics are not unique, but are always found in equations. In other words, these elements always exist in a specific context, so mathematics needs context to give results; therefore, the mathematical process in a sentence depends on the positioning of the words in a context, and not through its lexical existence.

Logically, a word forms a mathematical element located in a sentence; this happens by giving a symbol to each word, as follows:

Subject = A, predicate = B, object= C

We define how A relates to B, and how B relates to C, and how A relates to C; the relation between these elements will determine the mathematical relation in the sentence.

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The reproduction of words in the Arabic language takes place through derivation; this is done by adding certain letters to the root, and this addition increases the meaning of the word. In fact, the derivation process occurs through three elements: the additive letter, the root, and the new word; this process creates a linear function consisting of linguistic elements that have mathematical data. The process begins by determining the root value as a constant term, defining the additive letter as a variable, then indicating the rate of change between the new words obtained from the process of derivation; after which, the graph can be plotted.

MATERIAL AND METHODS

The rules that control the syntactical process require to collect the mathematical data in the sentence; this means we must indicate the limit of the sentence by indicating its initial word and its final word to create a finite set because it is possible to define all elements in an infinite set.

This study will attempt to define the mathematical value of linguistic structures that may be a word structure or a sentence structure; so the study does not aim to verify Arabic grammar, but it tries to explain the mathematical value inherent in the Arabic language; this is achieved through the process of analysis and proofing.

The analysis process begins from a specific problem that poses a group of questions:

1. How to determine the value of a linguistic root using mathematics?
2. How to achieve derivation as it occurs in language?
3. How to determine the mathematical relation in an Arabic sentence?

We know that the root is not an infinite value, so we can define its minimum and maximum value using equality; next comes the collection of data. The given data can be identified thanks to the human common sense and it does not need any proof. Then, we have the application of these data to the elements of mathematical rule. Thus the grammatical data are transformed into mathematical elements.

In Arabic, a word can be derived from another word; this happens when adding a specific letter to the grammatical root. First, we can add one letter, and then add two letters, and so on, so we can observe here a rate of change between the derived words; this rate of change is equal to the difference between the numbers of letters for each word. The data that are obtained upon root reproduction form the basis of the mathematical rule.

There are certain types of mathematical relations: symmetric relation, reflexive relation, transitive relation; every relation has specific characteristics that can be found within certain types of sentences. This is based on how words are connected to each other in the sentence.

RESULTS AND DISCUSSION

Arithmetic of Roots

The root in the Arabic language is an issue in mathematical grammar: It is the word from which the derivation process begins and it is a lexical unit consisting of a number of letters. The root is a finite value that consists of a specified number of letters, so the derivation of the root takes place in the meaning and pronunciation of the word because the word obtained from derivation carries a new meaning and a new pronunciation. It is noteworthy that the meaning of the word is not measurable while the pronunciation of the word can be measured.

The study of the root is one of the most important issues in grammar. Analyzing the value of a root begins with two problems:

Problem1: why do we need the root in language?
Problem2: what is the number of letters of the root?
Problem3: How does the derivation occur in the root?

Proof of problem 1

The process of derivation begins with a meaningful word because the word that does not carry a meaning cannot accept derivation. Consequently, any mathematical process that involves multiplying the letter with an empty word will not produce an answer, and the process of differentiation to produce words cannot begin from zero because the empty word is equal to zero, and multiplying any value by zero is equal to zero. This means that the word that does not have a meaning will always have a result equal to zero, for example:

بأ× ج = لا ناتج ؛ بأ ×ح = لا ناتج

zero × 1 = zero , zero × b = zero

Mathematically, the process of differentiation to produce words requires words that carry a meaning, and that have a tangible value with a finite number of letters with specific meaning, which will form an element in the mathematical rules.

Proof of problem 2

The root in the Arabic language has a numerical value between two and five letters, so the Arabic root is divided into three types: Binary root consisting of two letters, Triple root consisting of three letters, and Quadrant root consisting of four roots. The root has a numerical value that makes it a finite value, which can be easily measured, and it has a mathematical value as explained before. This means that the root has a value located in a mathematical inequality; this defines the minimum and the maximum value of the root (Zahoor, 2008). The Arabic root has a symbol “r”:

$$2 \leq \text{ Arabic root} \leq 5$$

Evidence 1

The root cannot be equal to one alphabetical letter because the root is a word, and it is made up of finite letters, so one letter does not equal to a word; therefore, the lower limit of the root consists of two alphabets.

Evidence 2

The root has the ability to derive; this process aims to produce new words from the root, and it is noted that the highest limit of the word during the derivation process consists of six letters. Knowing that there are no more than six letters in the morphological scales, if we add a letter to a root of six letters, we will get a word of seven letters, and this contradicts the principle of word construction. In other words, if the root is hexagonal and the maximum limit of the derivation process is equal to six letters, then the root cannot reproduce or move to a second grammatical value larger than six letters. Mathematically, we notice from the following data that the maximum limit of the grammatical root is equal to five alphabetic letters, and it cannot reach six or more letters.

If the local maximum of the root is equal to five, the derivation will be achieved.
If the local maximum of the root is equal to six, the derivation will not be achieved.

Linear function

Root “r” has the potential for grammatical reproduction through the addition of extra letters, so the root has a transmission motion from a previous word to another by derivation. The mathematical derivation depends on a specific value called the range x, that is the independent variable, which is grammatically the letter “l” because the letter changes from one word to another when adding specific letters to the root. When the independent grammar variable “l” changes, the variable x is achieved. The second element in this process is the root, this root is a constant value in all derived words, for example:

كَرُمَ،... أَكْرَمَ، ...، تَكَارَمَ،... اِسْتَكْرَمَ.

We observe that the root “*karoma*” is found in all the derived words, so this root is the constant term in this sequence of words. The third element in this process is the rate of change between the derived words. As seen in the previous example, every word has a finite number of letters, and the number of letters for every word increases from one word to another. The fourth element is the conclusion: in this process that is the derived word which happens when we add a specific letter to the root under a specified rate in every word. The grammatical elements have mathematical capabilities that allow them to be mathematical functions.

In linear function $f(x) = a + bx$

Element "a" is the constant term in linear function; we can explain this element grammatically whereby the constant term in the sequence is the root, because the root is present in all the derived words. Element "x" is the independent variable, so its value changes on x-axis, equivalent to that of the letter added to the word during the derivation process. Element "b" is the rate of change between the derived words; the difference between every word and another is one letter (table1), so the rate of change is equal to one. Finally, we get the result $f(x)$ or "y" that is the derived word "d".

Table 1. Subtraction between derived words is equal to one.

"6" letters – "5" letters = 1 استفعلَ – تفاعل = 1
"5" letters – "4" letters = 1 تفاعل – فاعلَ = 1
"4" letters – "3" letters = 1 فاعَلَ – فعَلَ = 1
Element a = r   element b = 1
x = independent value "added letter", "L" y = dependent value "derived word" "d" $f(x) = f(L)$

Grammar Mathematical Function

Words are generated when a specific letter increases in the quantitative content of the root, so that the process of moving from one word to another varies when the added letter is being added to the root. We have proved that this added letter is a variable independent element. This independent variable is located on the x-axis; when we apply the linear function rule, we can get the dependent variable "derived word" located on y-axis, so when the value of "x" or value of "l" changes the value of "y" changes. The relation between "x" and "y" is done under a rate of change "b", and thus a mathematical criterion for the grammatical derivation process can be formed based on two variables; this leads to an increase in the meaning of the derived words in a grammatical sequence, For example:

كَـــرُمَ،...، أكـــرَمَ،...، تـــكـــارَمَ،...، اســــــتــــكْـــرَمَ.

word3   word2   word1   Root

So the meaning of word3 is more supported than the meaning of word2 and so on.

In Arabic, the derivation of words has a specific data that can be transformed into mathematical elements following the linear function (figure1):

$f(x) = a + bx$ (Axler,2015), we prove that: $a = r$ and $x = l$ and $b = 1$

$$\Rightarrow f(x) = f(l) = a + b\times l = r + l \times 1 = r + l$$

Arabic example:

نعتبرُ أنَّ المتغيِّر الحرفي "ح" يســــاوي "ء" ، مِنْ هنـا فـإنَّ الـدالَّـة ســتســــاوي أفعَـلَ وذلـكَ يعودُ ل:

فاعَلَ = ا + جذر فعَلَ = *f*(ح)

نعتبرُ أنَّ المتغيَّر "ح" يســــاوي "ا.ت" ، مِنْ هنـا فـإنَّ الـدالَّـة ســتســــاوي فـاعَـلَ ذلـكَ يعودُ ل:

تفاعلَ = (ت. ا) + جذر فعَلَ = *f*(ح)

نعتبــرُ أنَّ الحــرف "ح" يســاوي "ا.س.ت" ، مِــنْ هنــا فــإنَّ الدالَّــة ستســاوي فاعَــلَ ذلــكَ يعــودُ ل:

استفعلَ = (ا.س.ت) + جذر فعَلَ = *f*(ح)

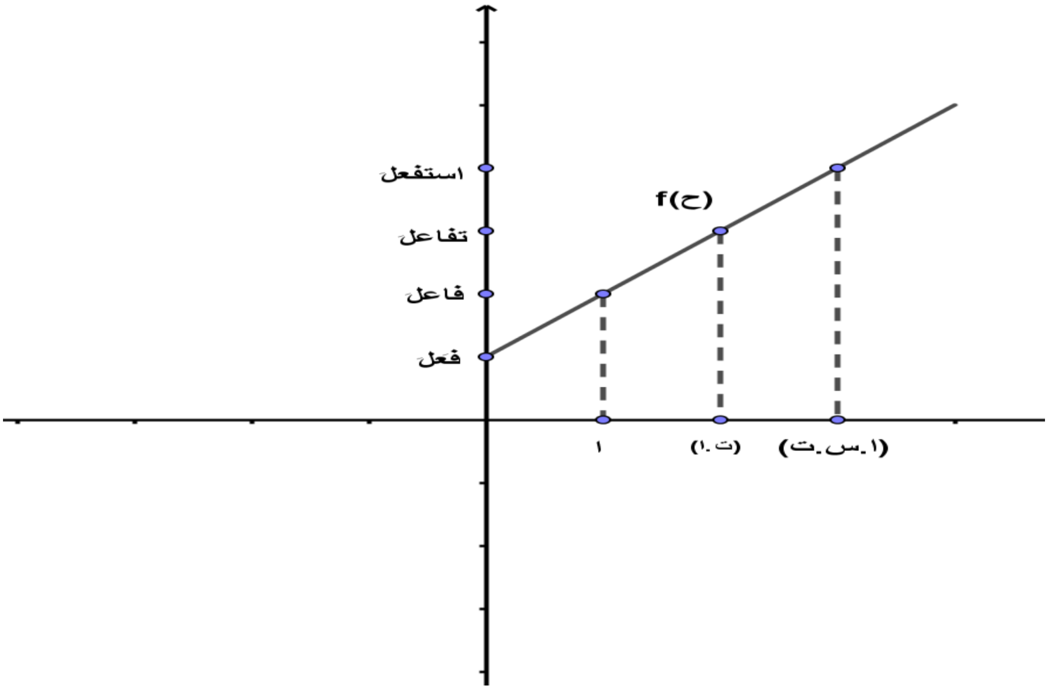

Figure 1. Grammatical linear function.

As shown in Figure 1, the derivation of words in the Arabic language takes place when the linear function is formed by two grammatical variables: the variable x, which is grammatically the variable “l”, and the variable “y” which is grammatically the variable “d” that forms the words generated from the triple root. It is achieved upon adding the value of a specific root with a new letter and multiplying it with the rate of change. This will produce a new word that is composed of a group of letters, which makes the quantitative content of the word. The result is a number of new words that have real meaning.

Permutation of a root

The probability of multiplying the added letter by a triple root will produce a finite number of possibilities; these possibilities are words that may have meaning or elements that are meaningless (Rušdï, 2010), as shown in (Table 2). The Permutation rule specifies the number of possibilities resulting from a set of four letters

$$^{N}P_{r} = n! \div (n-r)! \Rightarrow P = 4 \times 3 \times 2 \times 1 = 24 \text{ words}$$ (Brualdi,2009).

The first word in the table of these probabilities can locate on y-axis, because it has a meaningful, other words are meaningless:

Table 2. The probabilities of a word contain four letters.

(ع، ء، ل، ف)	(ع، ء، ف، ل)	(ء، ف، ل، ع)	(ء، ف، ع، ل)
(ع، ف، ل، ء)	(ع، ف، ء، ل)	(ء، ف، ل، ف)	(ء، ع، ف، ل)
(ع، ل، ف، ء)	(ع، ل، ء، ف)	(ء، ل، ع، ف)	(ء، ل، ف، ع)
(ل، ء، ع، ف)	(ل، ء، ف، ع)	(ف، ء، ل، ع)	(ف، ء، ع، ل)
(ل، ف، ع، ء)	(ل، ف، ء، ع)	(ف، ع، ل، ء)	(ف، ع، ء، ل)
(ل، ع، ف، ء)	(ل، ع، ء، ف)	(ف، ل، ع، ء)	(ف، ل، ء، ع)

The probability of multiplying two added letters by the triple root will produce a finite number of possibilities:

$$^{N}P_{r} = n! \div (n-r)! \Rightarrow P = 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ words}$$

One word of these probabilities can locate on y-axis, other words are meaningless:

The probability of multiplying three added letters by the triple root will produce a finite number of possibilities (Kolmogorov,1999):

$$^{N}P_{r} = n! \div (n-r)! \Rightarrow P = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \text{ words}$$

One word of these probabilities can locate on y-axis, other words are meaningless:

We have a four-letter word, which includes a repeated letter; from this, twelve possibilities arise; so the word that can be located possibility is a real word from every permutation rule:

$$^{N}Pr_{K} = n! \div [K!(n-k)!] \Rightarrow Pr = 4! \div [1! \times 2! \times 1!] = 24 \div 2 = 12$$

The first word of these probabilities can locate on y-axis, because it has a meaningful, other words are meaningless (Table 3).

Table 3. The probabilities of a word contain four letters, and one letter is repeated.

One real word, and three empty words	(ف، ع، ع، ل) (ف، ع، ل، ع) (ف، ل، ع، ع) (ع، ل، ف، ع)
Empty words, [meaningless words]	(ع،ع، ف، ل) (ع، ع، ل، ف) (ع، ل، ع، ف) (ع، ف، ل، ع)
Empty words, [meaningless words]	(ل، ع، ع، ف) (ل، ع، ف، ع) (ل، ف، ع، ع) (ع، ف، ع، ل)

Relation in the sentence

Words in a sentence are adjacent together; this feature requires a law that achieves the relation between words. How can this be determined by mathematics? There are many types of relation in mathematics. One of them is symmetric relation (Harrie de Swart, 2018) that can be applied in a sentence; this relation takes place when x relates to y, and y relates to x in the same relation, in arabic sentence, the subject is considered as x, and the predicate word as y (Table 4):

Table 4. The relation between two words in nominal sentence.

نورٌ القمرُ.	القمرُ نورٌ.
y x	y x
Subject Predicate	Predicate Subject

In the previous sentences, we notice that: x relates to y because all y is in x in the first sentence, and y relates to x in the second sentence because all y is in x, so x and y have the same relation in two forms: $x \Re y$, $y \Re x$; so we have: $\forall x, y, (x, y) \in \Re, (y, x) \in \Re$, as a result, the two words have a symmetric relation.

The process of constructing a word begins from a specific value that is the root; this value can be a mathematical element where the root and the derived words share common letters and a common meaning. The main feature of Semitic languages is that the meaning of the word is related to a specific root, and that the meaning in every word that is performed by specific letters does not deviate from the main meaning; so the words in a grammatical sequence share common letters and common meaning. The root shares a certain number of letters with the derived words, noting that the meaning increases when the numerical value of the letters increases in the word. However, the process of differentiation is not completely independent but related to the value of the root because the root is the common unit in a sequence of words. We notice that in the graph of a linear function, all values that lie on "y" contain the three letters of the root. The intersection law can prove this Phenomena in Semitic language (Figure 2), (Table 5).

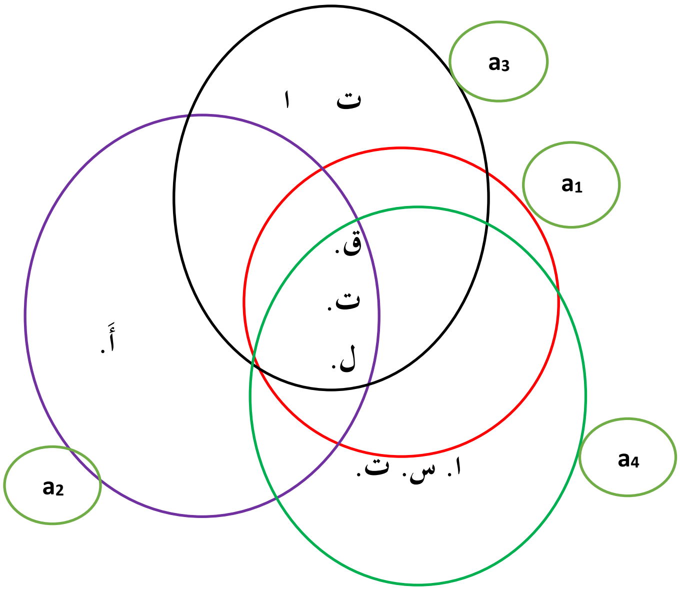

Figure 2: intersection between derived words

Table 5. The intersection between derived words.

Set a_1 = قَتَلَ
Set a_2 = أَقْتَلَ
Set a_3 = تَقَاتَلَ
Set a_4 = اسْتَقْتَلَ
Set $a_4 \cap$ Set $a_3 \cap$ Set a_2 = Set a_1
Set a_1 is the root of the derived words

CONCLUSION

This study has shown the mathematical content of the grammatical issues. These issues may revolve around the morphological structure or syntactic structure; this happens when we indicate the mathematical criteria that control the processes of building Arabic sentences, thus securing the correct construction of the grammatical sentences. Here, the study determines through mathematics the derivation rules for Arabic words. Research examines words as mathematical elements of context, context is a key factor in studying words.

Mathematically, the function is a mathematical process that occurs from two variables; the first variable is an independent element and the second is a dependent element, so the movement of the second variable follows the movement of the first variable. This happens when words are derived, and through this process, we transform the roots and derivative words into mathematical elements.

The words resulting from the derivation process form a mathematical sequence; it is a series of words related to the roots where these derived words connect the root with specific letters. We can therefore perform the intersection law in order to determine the value of the root from a group of words belonging to a sequence of words.

The calculation of Permutation rule determines the actual probabilities that can form the real dependent variable from a set of neglected probabilities that do not carry any specific meaning.

Finally, the mathematical work is not limited. On one hand, the mathematical standards reveal the temporal content of the verb through certain types of functions as the continuous and constant function; this becomes true by calculating the time dimension of the verbs. On the other hand, the mathematical relation in the sentence can be reflexive or transitive; consequently, the grammar issues are studied through mathematics. The Arabic language is subject to the rules of thought through the tools of mathematics. This study is an attempt to bring Arabic closer to the technology of the era, especially computer programming.

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